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Dynamics of Information in Distributed Decision Systems

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Abstract

This paper considers distributed decision systems exemplified by subdivisions competing for corporate resources. Corporate coordinators oversee the otherwise independent activities of the subdivisions by setting prices (or premiums) on the resources. Taking such prices into account, subdivisional planners submit proposals which will be weighed by the coordinators who then may adjust the prices. If equilibrium prices can be established, an optimal allocation of the resources will be made by the coordinators. The behavior of this decision process is greatly affected by the manner in which information is communicated among the parties involved. We discuss four classes of information schemes and study the dynamics of information involved. It is shown that the overall effectiveness of any scheme depends on behavioral patterns of the interacting agents and that no single scheme can be universally superior. However, it may be possible to identify favorable schemes for specific decision systems by observing patterns in the behavior of the coordinators and the planners.

Keywords: Distributed Decision Systems; Information Management; Parallel Processing;
Linear Programming Decomposition.

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1. Introduction

Practically all systems and organizations are structured with interacting components. Regardless of whether there is a clear-cut hierarchy of commands or not, important global decisions invariably involve the judgment made locally at the component level. We call such an environment a distributed decision system. Central to its successful operation is the process of coordination which always relies on the communication of information among the components, regardless of the underlying decision making policy. When such coordinating information becomes available and when it is actually used will be important factors in the effectiveness of the system. We call the study of these factors the dynamics of information in distributed decision systems.

To study and analyze the dynamics of information in the broadest sense of distributed decision systems will be difficult, if not impossible. As a first step, we narrow our attention to a specific and somewhat more tractable situation which can be modeled as the price-directive decomposition of a block-angular linear program (see e.g. Dantzig 1963, Jennergren and Müller 1973, Maier and Vander Weide 1976, Christensen and Obel 1978, Dirickx and Jennergren 1979, Nazareth 1980, Samouilidis and Arabatzi-Ladia 1984). Consider a corporate structure with subdivisions competing for common resources. Corporate coordinators oversee the otherwise independent activities of the subdivisions by setting prices (or premiums) on the resources. Taking such prices into account, subdivisional planners submit proposals which will be weighed by the coordinators who in turn may adjust the prices. If equilibrium prices can be established, an optimal allocation of the resources will be made by the coordinators. Assuming further that both the objective, corporate resource and subdivisional operations can be modeled by linear constraints, the above decision process is exactly the Dantzig-Wolfe decomposition algorithm for the underlying linear programming model. Note that we are not proposing LP decomposition as a decision process. Rather, we assume that certain real life decision processes can be approximated in such a way and proceed to analyze their dynamics of information. This is made possible by studying various information schemes that correspond to different solution strategies in the decomposition algorithm.

After a brief review of Dantzig-Wolfe decomposition of block-angular linear programs, four information schemes which determine the precise flow of proposals and prices (i.e. dynamics of information) in the process of coordination are described. A numerical example is then used to illustrate the efficacy of these schemes. It is shown that the relative effectiveness of the information schemes under study depends on inherent properties of the system, e.g. the relative complexities of the coordinating and subdivisional problems. For this reason, it would not be appropriate to search for a universal scheme that is superior for all decision systems. However, we may be able to identify the most effective scheme for a given system by a priori analysis. In this sense, we can classify distributed decision systems by their dynamics of information.

2. Dantzig-Wolfe Decomposition of Block-Angular Linear Programs

2.1 Summary of the Algorithm

As a first step in analyzing the dynamics of information in distributed decision systems, we consider decision processes that can be modeled as Dantzig-Wolfe decomposition (Dantzig and Wolfe 1960) of a block-angular linear program (BLP). Such a problem with R blocks has the following form in which vectors and matrices are denoted in bold-face, and a prime denotes a transpose.

$$(P) \quad \text{minimize } \sum_{r=0}^{R-1} c_r' x_r \quad (1)$$

$$\text{subject to } \sum_{r=0}^{R-1} A_r x_r = b_0 \quad (2)$$

$$B_r x_r = b_r; \quad r = 1, \dots, R \quad (3)$$

$$x_r \geq 0; \quad r = 0, \dots, R \quad (4)$$

where c_r is $1 \times n_r$, b_r is $m_r \times 1$ and all other vectors and matrices are of compatible dimensions.

First, we summarize the Dantzig-Wolfe decomposition algorithm (see Ho 1987 for recent development in this approach). Then its interpretation as a distributed decision process will be given. Let $F_r = \{x_r \mid B_r x_r = b_r, x_r \geq 0\}$ be the set of all feasible solutions to subproblem r , and $X_r = \{x_r^k; k=1, \dots, K_r \mid x_r^k \text{ is an extreme point of } F_r\}$ be the set of extreme points of F_r . For simplicity, we assume that F_r is nonempty and bounded. It is then a bounded polyhedral convex set. Using the fact that any point in such a set can be represented by a nonnegative convex combination of its extreme points, one can rewrite (P) in the following equivalent extremal form [Dantzig 1963]:

$$(E) \quad \begin{aligned} & \text{Min } \sum_{r=1}^{R-1} \sum_{i=1}^{K_r} (c_r' x_r^i) \lambda_{ri} \\ & \text{s.t. } \sum_{r=1}^{R-1} \sum_{i=1}^{K_r} (A_r x_r^i) \lambda_{ri} = d \\ & \quad \sum_{i=1}^{K_r} \lambda_{ri} = 1; \quad r=1, \dots, R \\ & \quad \lambda_{ri} \geq 0, \quad i=1, \dots, K_r; \quad r=1, \dots, R, \quad \text{where } x_r^i \in X_r. \end{aligned}$$

Thus solving (E) is equivalent to solving (P). Since K is often very large and not known a

priori, a relaxation strategy is applied to solve (E). The Dantzig-Wolfe decomposition algorithm uses a subset of J_r extreme points in X_r to formulate a Restricted Master Problem, say,

$$\begin{aligned}
 & \text{Min } z^k = \sum_{r=1, \dots, R} \sum_{i=1, \dots, J_r} (c_r x_r^i) \lambda_{ri} && \text{Dual Variables} \\
 & \text{s.t. } \sum_{r=1, \dots, R} \sum_{i=1, \dots, J_r} (A_r x_r^i) \lambda_{ri} = d && \pi^k \\
 (M^k) \quad & \sum_{i=1, \dots, J_r} \lambda_{ri} = 1; \quad r=1, \dots, R && \sigma_r^k \\
 & \lambda_{ri} \geq 0, \quad i=1, \dots, J_r; \quad r=1, \dots, R.
 \end{aligned}$$

During the k -th cycle of the algorithm, (M^k) is solved. Let $(\pi^k, \sigma_1^k, \dots, \sigma_R^k)$ be the optimal dual solution. The vector π^k is known as the *price vector* corresponding to the coupling constraints. To see if the objective can be further improved by introducing any extreme point in X not yet included in M^k , the following subproblem is solved. This is essentially an implicit simplex pricing step using the prices π^k .

$$\begin{aligned}
 & \text{min } v_r^k = (c_r - \pi^k A_r) x_r - \sigma_r^k \\
 (S_r^k) \quad & \text{s.t. } B_r x_r = b_r \\
 & x_r \geq 0.
 \end{aligned}$$

If $v_r^k \geq 0$, then nothing in F_r can improve the objective. If this holds for all r , then the solution of (M^k) is optimal for (E). Otherwise a new extreme point, called a proposal, with $v_r^k < 0$ can be included in (M^{k+1}) for an improved solution in the next cycle. Convergence is finite as each X_r is a finite set (see [Dantzig 1963]). Furthermore, if ρ^k is the dual optimal solution to (S_r^k) , then $(\pi^k, \rho_1^k, \dots, \rho_R^k)$ is a dual feasible solution to (P) and hence provides a lower bound z^k on the minimum value of z . Therefore, in practice, we can stop the decomposition algorithm if $|z^k - z^k| < \epsilon$ for some $\epsilon \geq 0$.

2.2 A Distributed Planning Scenario

The above algorithm can be interpreted as a distributed decision system. Note that our purpose is not so much proposing such a process for decision making as assuming that it is the appropriate model, perhaps as a first approximation, for certain planning scenarios. Consider a planning process where there are R planners representing as many divisions of a company. Each planner has at his disposal a certain amount of local resources and shares some company wide resources with other planners. As the planning process proceeds, each planner is required to submit local plans (proposals) by solving a local subsystem (an LP). Each proposal submitted must improve the current objective value of the entire planning problem. Therefore, a proposal represents a local decision that specifies the amount of shared resources required for its implementation.

A *coordinator* is put in charge of the process so that an optimal plan for the entire problem can be obtained. He does this by controlling the allocation of shared resources. Each of the shared resources is given a price, which represents its worth. The planner will be charged for using the shared resources. Based on proposals submitted by the planners, the coordinator will determine a set of prices for the shared resources by solving a *Master Problem*, which is an LP formulated using all the proposals on hand. The new prices will then be sent to the planners. The planners discard the old prices and work on new plans according to the new prices. This iterative process continues until the coordinator arrives at a set of prices at which no new proposal is generated by the planners. At this point, the planning process is said to have converged to optimality. The information flow in this planning scenario is shown in Figure 1.

The rate at which the planning process converges to optimality is of basic interest. Clearly, the proposals and prices information is crucial to the dynamics of the above planning process. Since the coordinator is working concurrently with the planners, the set of prices for the shared resources will be constantly changing as new proposals are being incorporated by the coordinator. For the coordinator, the questions are: when should the new prices be sent to the planners? Which prices should be sent? When should the planners be told to stop submitting proposals according to certain prices? The generation of proposals by each planner is very much influenced by when and what prices are received from the coordinator. In turn, the timing and nature of the proposals affect directly the prices generated by the coordinator. For the planners, the questions are: when, and which proposals should be submitted? When should a new set of prices be used in generating proposals? Previous studies of price-directive decomposition (see e.g. Burton and Obel 1977 and 1980, Moore 1979) as well as the dual approach of resource-directive decomposition (see e.g. Silverman 1972, Ten Kate 1972, Freeland and Moore 1977, Burton and Obel 1980) emphasized primarily on the organizational structure and the hierarchy of information flow among the agents. The present work focuses on the concurrent nature of the activities of the agents as well as the timing and communication of information among them. The algorithmic implications are found to

be significant in the advent of parallel computation using multiple-processor computers (see Ho et al 1988).

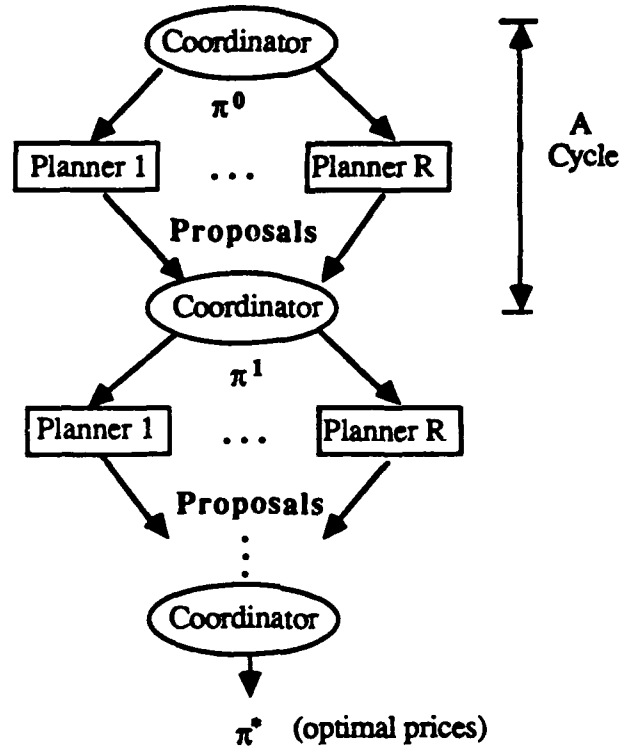


Figure 1. Information Flow in Dantzig-Wolfe Decomposition

3. Dynamics of Information in Distributed Dantzig-Wolfe Decomposition

The timing and nature of information made available and utilized in the above planning process is the subject of our study. Several schemes that control the flow of information will be described in the following sections. The dynamics of information and its effect on the decision process will be discussed. It should be remarked that while the interpretation of LP decomposition as a distributed decision process (or what is traditionally termed decentralization) is well known, our approach, which takes into account of concurrency, did not become obvious until the recent advent of parallel computation. To make the distinction algorithmically, we shall refer to implementations of LP decomposition which take advantage of parallel computation as distributed Dantzig-Wolfe decomposition.

An information scheme in distributed Dantzig-Wolfe decomposition controls the flow of proposal and prices information. We consider four information schemes, namely, the Basic Information Scheme (BIS), the Early Start Information Scheme (ESIS), the Early Termination Information Scheme (ETIS), and the Intermediate Prices Information Scheme (IPIS). They are compared in terms of synchronization, information flow and concurrency.

3.1 Basic Information Scheme (BIS)

In a decision system corresponding to the sequential Dantzig-Wolfe decomposition algorithm (SDWDA), the R planners and the coordinator can be considered to be sharing a single information processing facility. They take turn using the facility. Once a new set of prices is announced by the coordinator, each planner queues up to solve his subsystem and stores the proposals in a common buffer used by all the planners. When all the planners have completed their proposals generation, the coordinator will collect all the proposals from the buffer to determine the next set of prices.

In a scenario corresponding to the distributed Dantzig-Wolfe decomposition algorithm (DDWDA), the coordinator and planners each has his own information processing facility and can process the information in parallel. For the purpose of comparison, we consider a scheme for DDWDA where the flow of information is fully synchronized and identical to SDWDA. We call this the Basic Information Scheme (BIS). BIS differs from SDWDA in that the planners will generate proposals concurrently, while the coordinator will be idle and waiting for all the proposals to come in. The process of determining new prices will begin after all the planners have submitted their proposals. During the time when the coordinator is processing the proposals to get a new set of prices, all the planners will be idle and waiting for the new prices to start another round of proposals generation.

An illustration of this information scheme with three planners is shown in Figure 2. In this and the following figures, the black rectangles are used to denote the busy time of the coordinator,

and the shaded rectangles the busy time of the planners. The white areas denote idle time.

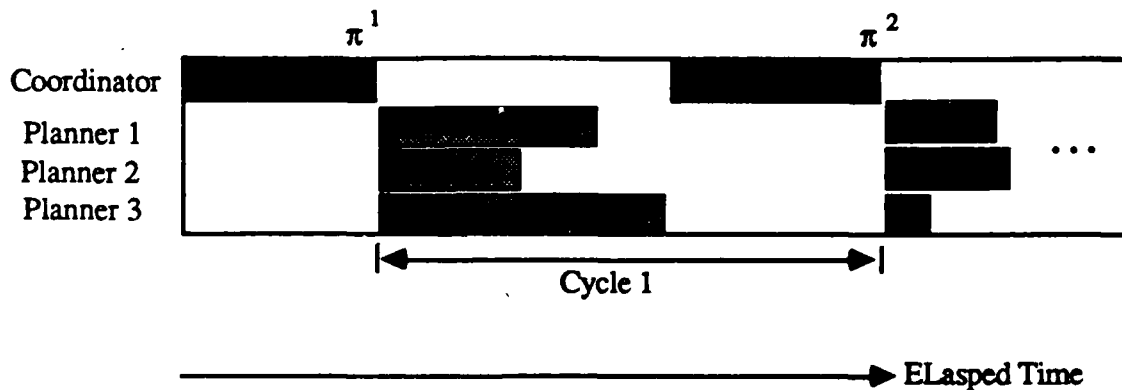


Figure 2. Basic Information Scheme (BIS)

3.2 Early Start Information Scheme (ESIS)

In BIS, the coordinator is totally unproductive when the planners are working. Actually he can begin processing some of the proposals as soon as they have been submitted. Note that eventually, he will have exactly the same proposals for a particular cycle as in BIS. The only difference is that the coordinator starts his price setting problem earlier. We call this the early start information scheme (ESIS).

The early processing of proposals to determine new prices does not guarantee solving the Master Problem faster. It may even require more time than starting the process after *all* the proposals are received. However, it is hoped that the overall *cycle time* would be shorter due to increased concurrency among the coordinator and planners. Note that the proposals and prices generated in ESIS are identical to those in BIS. Therefore, any effect that ESIS may have on the elapsed time of the planning process is not due to the information but the early processing of the proposals. An intuitively typical case of ESIS is illustrated in Figure 3. Comparing Figures 2 and 3, we see that although the time required to generate π^2 in ESIS is longer than that in BIS, the overall time for cycle 1 is shorter. It is of course conceivable that the early start somehow leads to a much longer solution path to π^2 so that the cycle time turns out to be longer than that in BIS.

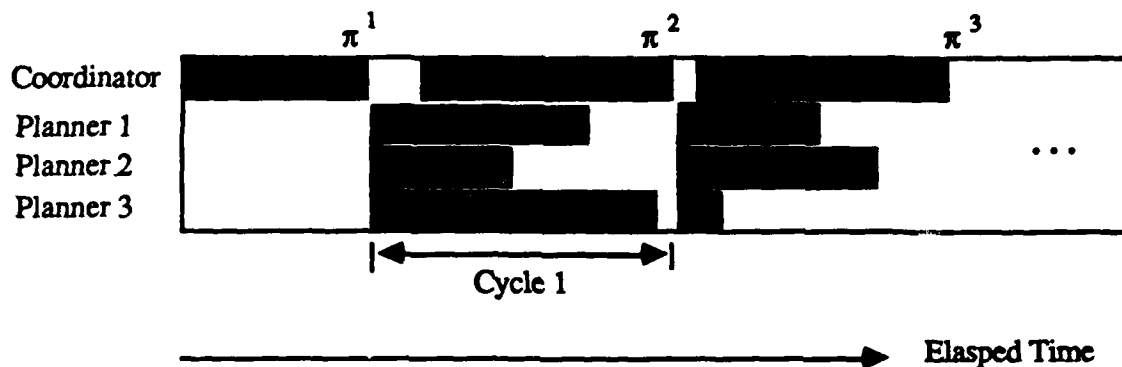


Figure 3. Early Start Information Scheme (ESIS)

3.3 Early Termination Information Scheme (ETIS)

Given a set of prices, each planner may take a different amount of time to generate all his proposals. In BIS, the planners who finish early will have to wait for other planners who are still working. In the early termination information scheme (ETIS), the coordinator orders *all* the planners to stop submitting proposals at some time t_x to reduce waiting times. The coordinator will then use the proposals that are generated prior to t_x to determine a set of new prices. The time t_x can be determined in several ways.

i) First Subproblem

Let t_x be the first time when a subproblem is completed. If there is at least one proposal among all the planners, they are asked to stop at t_x . Otherwise, they are allowed to continue until the first proposal is submitted.

ii) First Proposal

The planners' cycle is terminated as soon as the first proposal is submitted.

iii) Ad hoc

In principle, the coordinator can decide on when he should stop the planners depending on their proposals generation patterns. For example, he might observe a very uneven proposals generation pattern among the planners where one or two of them generate a large number of proposals while the rest generate very few and stop very early. In such a situation, he might allow fewer number of proposals to be generated by each planner. Similarly, he might increase the number of proposals allowed in later cycles when the proposals generation pattern changes.

ETIS is illustrated in Figure 4 . Note that although ETIS is depicted to have a shorter cycle 1 than BIS, the π^2 generated may be different. Intuitively, we expect ETIS to require more cycles than BIS since fewer proposals are submitted during each cycle. The advantage of ETIS is that all planners work an equal amount of time.

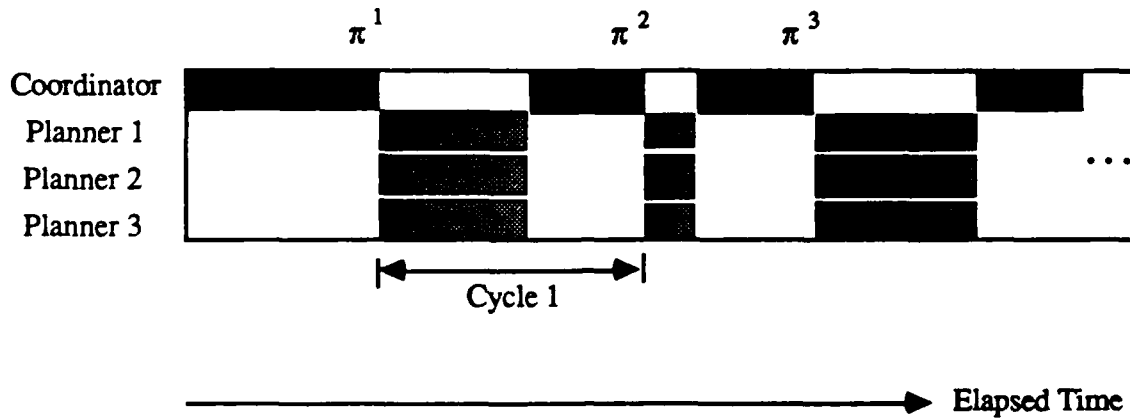


Figure 4. Early Termination Information Scheme (ETIS)

3.4 Intermediate Prices Information Scheme (IPIS)

The prices on the shared resources change as new proposals are processed by the coordinator. In all the information schemes described so far, this new information is not made available to the planners. Since the prices information is critical to the planners in deciding on which proposals to submit, any changes in prices should in principle be fed back immediately to the planners. Given a quicker feedback of price information, the planners should be able to make better decision on the consumption of the shared resources and submit proposals that would enable the planning process to converge faster. The question is which prices should be made available to the planners and when the planners should make use of these new prices.

Prices that are made available to the planners while they are still in the process of generating proposals are referred to as *intermediate prices*, and the information scheme that uses them is known as the *Intermediate Prices Information Scheme (IPIS)*. Intermediate prices determined by the coordinator can be fed back to the planner at different rate. Several variations of IPIS with different frequency of prices feedback are described below.

i) Instant Feedback

Any change in prices is immediately announced to the planners who will continue to generate proposals using the updated prices. This extreme utilization of prices information is called an *Instant Feedback Strategy*. Suppose the coordinator uses the simplex method to determine the prices. Then he will send out the new prices and incorporate new proposals into the master problem

after each simplex iteration. This information scheme is the closest to instant availability and utilization of information in a planning process. However, many systems constraints such as the capacity of the planners and the coordinator to store and process such intensive flow of information should be considered in the realization of this information scheme.

ii) *N-proposals*

To study how the different rates of prices feedback affect the behavior of the planning process, we control the amount of intermediate prices sent to the planners according to the number of new proposals that have been incorporated. This indirectly controls the "closeness" between two consecutive sets of prices. In an *N-proposals* IPIS, the coordinator does not process new proposals while he is computing new prices. New proposals submitted during this time will be stored in a buffer. Once a set of new prices is determined and released to the planners, the coordinator will then check the buffer for new proposals and collect up to *N* new proposals to determine the next set of prices. If there are less than *N* new proposals in the buffer, all will be collected and processed.

The frequency of prices sent to the planners can be regulated by varying the value of *N*. Increasing *N* tends to reduce the number of intermediate prices sent to the planners. For *N* = 1 we have the *One-proposal* IPIS; and for *N* = ∞ , we have the *All-proposals* IPIS.

IPIS is illustrated in Figure 5. Note that there is no clear cycles in this information scheme. In the figure, all the planners update the prices at time t_0, t_1, \dots, t_6 . A planner becomes idle if all proposals according to the current prices have been submitted and no new prices are received (e.g. planner 1 between t_3 and t_4).

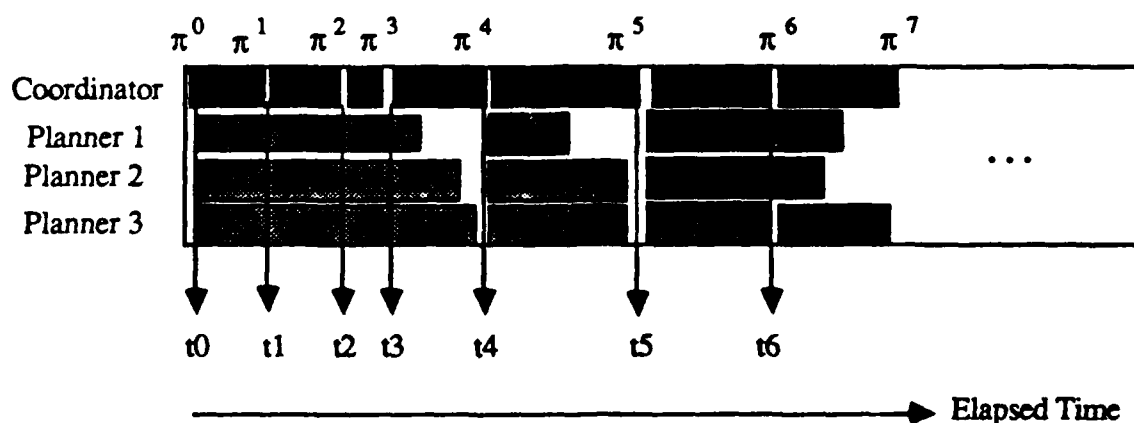


Figure 5. Intermediate Prices Information Scheme (IPIS)

3.5 Classification and Comparison of Information Schemes

Each of the four schemes described above can accommodate further strategic variations such as the exact quota on proposals for each master cycle. Therefore, one can begin to classify a whole spectrum of detailed decision-making policies into the four categories: BIS, ESIS, ETIS and IPIS. Before examining the potential effectiveness of the different schemes as distributed decision processes, we can introduce two attributes which provide insight into their behavior. The first is concurrency, i.e. the extent to which parallel processing is possible among the agents in the decision system. The second is information flow, i.e. the amount of communication required among the agents. Intuitively, increased concurrency should improve the convergence rate whereas increased information flow should have the opposite effect. The four categories of information schemes are mapped on these relative scales in Figure 6.

ESIS can be made to follow exactly the same path as BIS. Therefore, the information flow is the same for the two schemes. However, ESIS gains concurrency of the processes by letting the master (coordinator's) problem become active as soon as proposal information is made available. Therefore, one can expect ESIS to be more effective than BIS. In ETIS, concurrency among the planners is improved by reducing the time spent by the faster ones waiting for the slower ones. This policy may degrade the quality of proposals in a cycle. The overall effectiveness will be a trade-off dependent on other properties of the decision system. Similarly, variations of IPIS tend towards a process of continuous updating of information keeping all the agents busy. In so doing, the volume of communication is bound to increase. Whether it outweighs the benefits of keeping the agents better informed is again a matter of trade-off. We will show by numerical examples that each information scheme may be effective for some decision systems but not for others. For this reason, rather than looking for a universally superior scheme, it will be more meaningful to characterize distributed decision systems by their most favorable schemes.

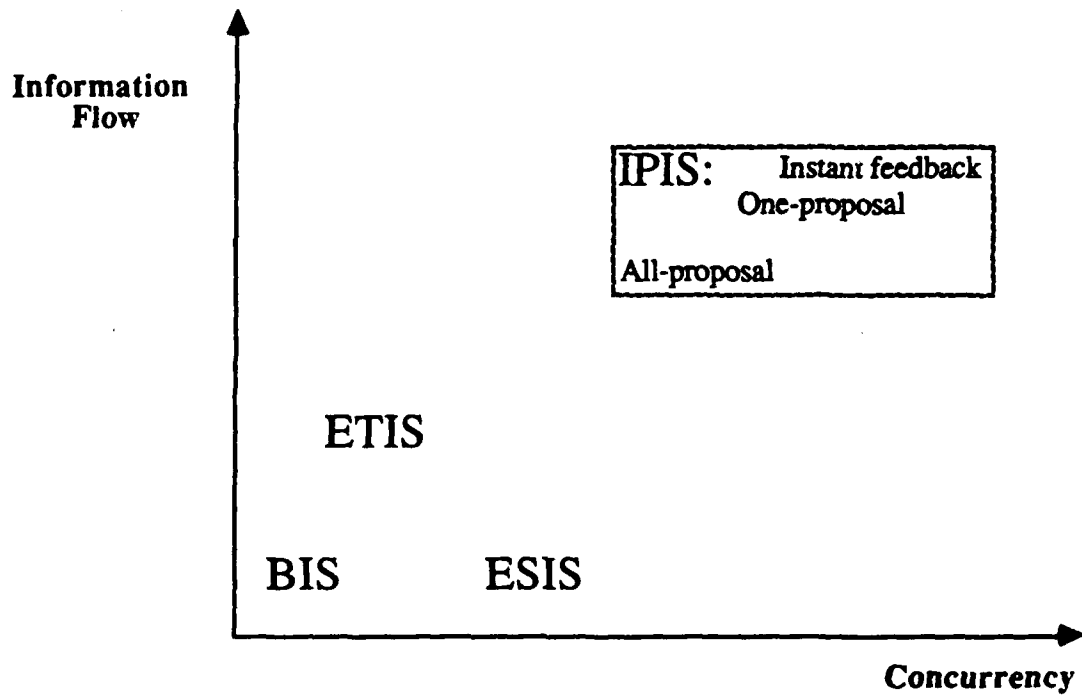


Figure 6. A Two-Attribute Comparison of Information Schemes

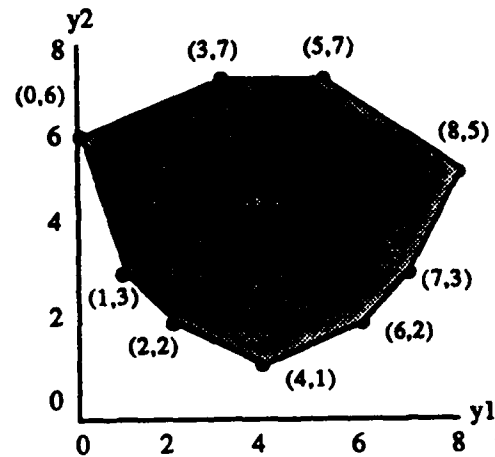
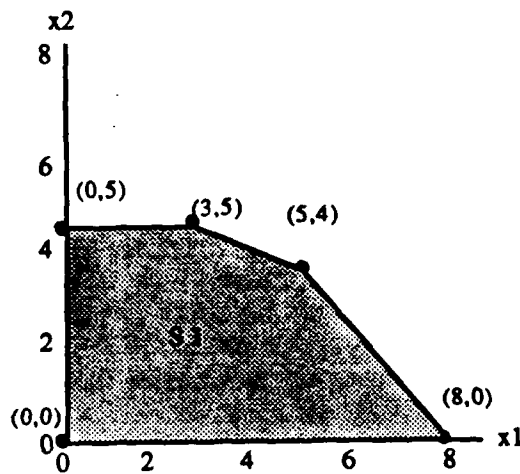
4. Numerical Examples

In this section, we illustrate the various information schemes in distributed DWDA with numerical examples. The examples are constructed to demonstrate the effect of different proposals generation patterns on the behavior of the algorithm. The following linear program is used throughout.

$$\begin{array}{llllllll}
 \min & -2Z_0 & +2Z_1 & & -X_2 & -Y_1 & +Y_2 & \\
 \text{s.t.} & & -Z_1 & -X_1 & & +Y_1 & & \leq 0 \\
 & Z_0 & & & +X_2 & & -Y_2 & \leq 0 \\
 & & & & & & & \\
 & & & & X_2 & & & \leq 5 \\
 & & X_1 & +2X_2 & & & & \leq 13 \quad (S1) \\
 & 4X_1 & +3X_2 & & & & & \leq 32 \\
 & & & & & & & \\
 & & & & & 3Y_1 & +Y_2 & \geq 6 \\
 & & & & & Y_1 & +Y_2 & \geq 4 \\
 & & & & & Y_1 & +2Y_2 & \geq 6 \\
 & & & & & Y_1 & -2Y_2 & \leq 2 \\
 & & & & & Y_1 & -Y_2 & \leq 4 \quad (S2) \\
 & & & & & 2Y_1 & -Y_2 & \leq 11 \\
 & & & & & 2Y_1 & -3Y_2 & \leq 31 \\
 & & & & & & Y_2 & \leq 7 \\
 & & & & & -Y_1 & +3Y_2 & \leq 18
 \end{array}$$

$$Z_0, Z_1, X_1, X_2, Y_1, Y_2 \geq 0$$

The feasible regions, sets of extreme points, and the corresponding proposals for S1 and S2 are given in Figure 7. In the following examples, the initial price vector $[\pi_1, \pi_2, \sigma_1, \sigma_2]$ is set to $[0 \ 0 \ 0 \ 0]$, and the initial set of proposals, Q^0 , consists of p_1 and q_6 .



We have: $C1 = [0 \ -1]$ and

$C2 = [-1 \ 1]$

Also,

$$A1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and}$$

$$A2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

For $S1$:

$$\begin{aligned} p1 &= [-5 \ 10 \ 5 \ 10] \\ p2 &= [-5 \ 1 \ -3 \ 5 \ 10] \\ p3 &= [-4 \ 1 \ -5 \ 4 \ 10] \\ p4 &= [0 \ 1 \ -8 \ 1 \ 10] \\ p5 &= [0 \ 10 \ 0 \ 10] \end{aligned} ; \quad \text{and for } S2 :$$

$$\begin{aligned} q1 &= [6 \ 10 \ -6 \ 0 \ 1] \\ q2 &= [2 \ 1 \ -3 \ 0 \ 1] \\ q3 &= [0 \ 12 \ -2 \ 0 \ 1] \\ q4 &= [-3 \ 14 \ -1 \ 0 \ 1] \\ q5 &= [-4 \ 16 \ -2 \ 0 \ 1] \\ q6 &= [-4 \ 17 \ -3 \ 0 \ 1] \\ q7 &= [-3 \ 18 \ -5 \ 0 \ 1] \\ q8 &= [2 \ 15 \ -7 \ 0 \ 1] \\ q9 &= [4 \ 13 \ -7 \ 0 \ 1] \end{aligned}$$

Figure 7. A Numerical Example

For ease of illustration, we assume that the time needed to solve the master problem is given by $t_0 = |Y|$ units, where Y is the set proposals incorporated into the master problem, and $|Y|$ is the cardinality of Y . Also, for simplicity, we assume no transmission time in sending messages. In all cases considered, a maximum of two proposals, the first and the last extreme points that qualify as proposals will be submitted by a subproblem in any cycle. For example, suppose $(4,1) - (6,2) - \underline{(7,3)} - \underline{(8,5)} - \underline{(5,7)}$ is the simplex path of extreme points followed by S2 with prices π^k , where the extreme points that qualify as proposals are underlined. Then only the proposals corresponding to $(7,3)$ and $(5,7)$ are generated.

The prices-proposals coordination process is shown in the form of a *prices-proposals graph* (see e.g. the top part of Figure 8). A square node in the graph represents π^k , where k denotes the order in which the prices are generated. A circular node denotes a proposal. A proposal q is generated with π^k if there is a directed arc from node π^k to node q . The price vector π^k is generated with the proposal set Q , which consists of all the q 's that have directed paths to the node π^k . A dashed arc from a node π^i to another node π^j indicates that all proposals used in generating π^i are also used in generating π^j .

To show the effect of different proposals generation patterns we use hypothetical solution times for the subproblems. This is simply equivalent to assuming various response times for the planners so that the results should represent plausible rather than overly contrived scenarios.

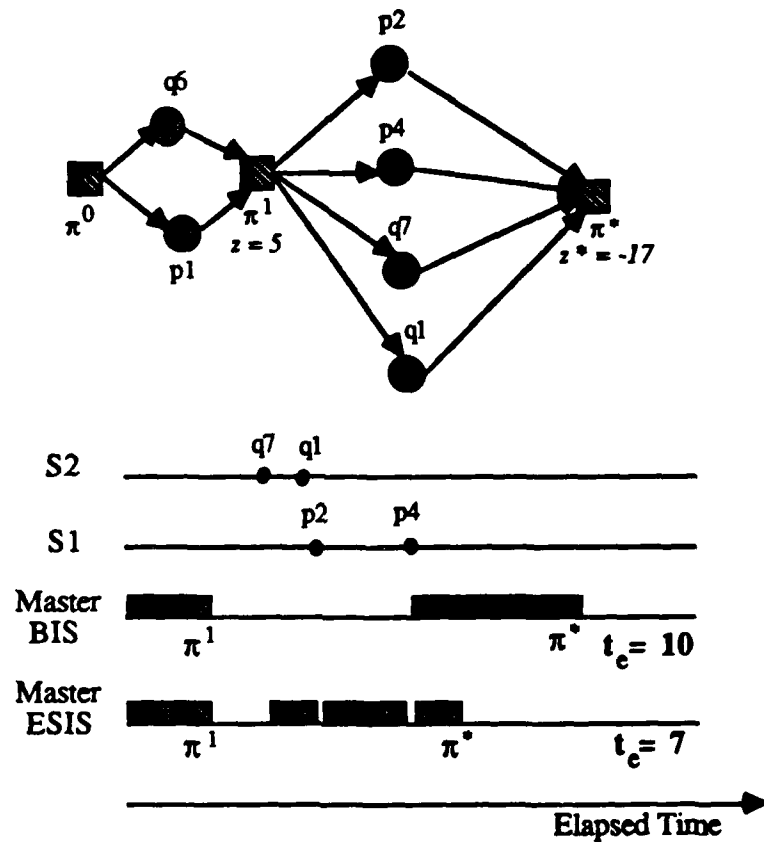
4.1 Examples of BIS and ESIS

Figure 8 shows the sequence of prices generated using BIS and ESIS. In this and subsequent diagrams, a black dot on the time line denotes the point in time when a proposal is generated. The shaded rectangles denote the busy periods when the coordinator is solving the master problems. With the hypothetical solution times given in Figure 8, the elapsed times for BIS and ESIS are 10 units and 7 units, respectively. Note that the elapsed time for BIS depends only on when the last proposal is generated in each cycle.

4.2 Examples of ETIS

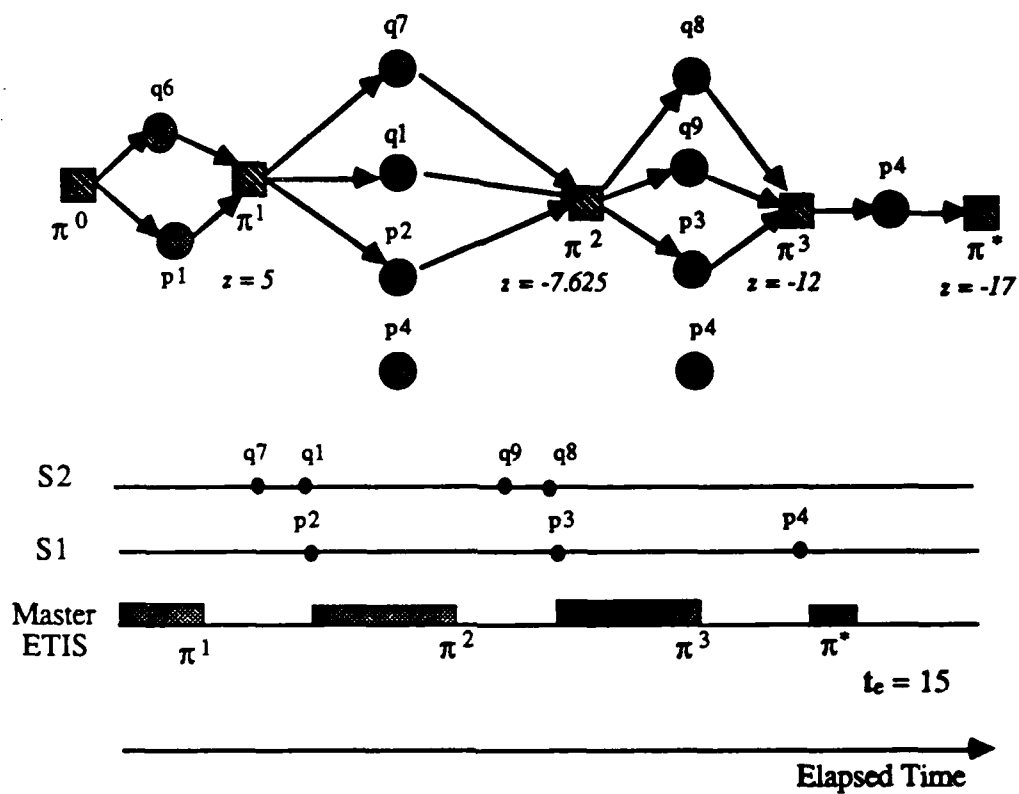
The prices-proposals graph using ETIS (First-subproblem) is shown in Figure 9. Comparing to the case of BIS, the omission of proposal p_4 in cycle 1 due to early termination results in the price vector $(-0.875, -2, 2.375, -6)$ in cycle 2. This causes the proposals p_3, q_8, q_9 to be generated. These proposals are not generated in BIS and ESIS. In this example, p_4 is a "critical" proposal in the sense that any basis without p_4 will not be optimal. The omission of p_4 leads to a longer sequence of prices and extraneous proposals. The reduction in waiting times (waiting for p_4 in cycles 1 and 2) is outweighed by the additional computational efforts required by a longer sequence of prices.

Figure 10 shows the behavior of ETIS using a different hypothetical proposals generation pattern. In this case, q_1 is omitted due to early termination. However, it does not cause a longer sequence of prices and actually provides an improvement in the elapsed time. Note that this gain comes from both the reduction of waiting times and a smaller master problem. The two examples of ETIS show that the relative response times of the planners are crucial factors in its effectiveness. If S1 finishes first, we have the second example and ETIS works well.



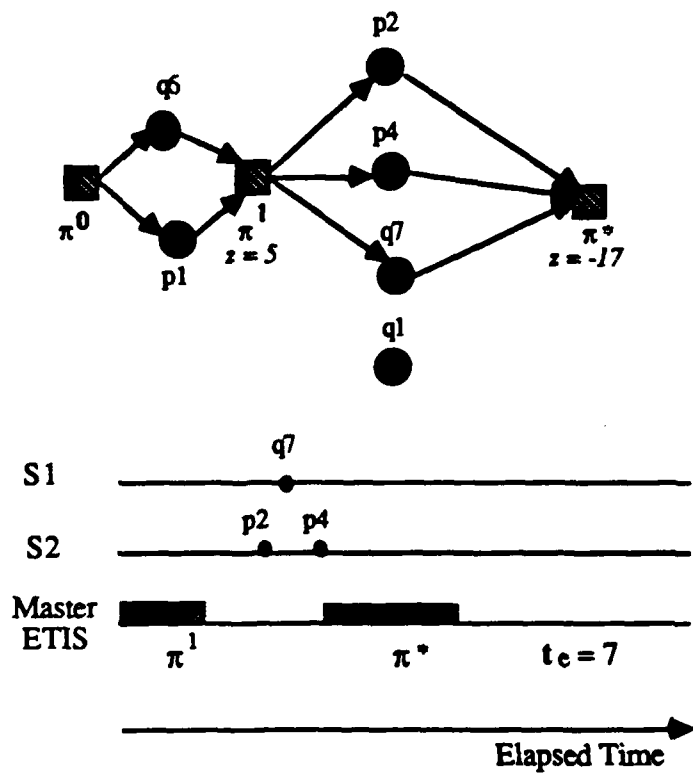
k	π	σ_1	σ_2	$(c_1 - \pi A_1)$	Q1	$(c_2 - \pi A_2)$	Q2
0	(0, 0)	0	0	(0, -1)	p1	(-1, 1)	q6
1	(-2, -2)	5	4	(-2, 1)	p2, p4	(1, -1)	q7, q1
2	(0, -2)	0	-13	(0, 1)	None	(-1, -1)	None

Figure 8. Examples of BIS and ESIS



k	π	$\sigma 1$	$\sigma 2$	$(c1-\pi A1)$	Q1	$(c2-\pi A2)$	Q2
0	(0, 0)	0	0	(0, -1)	p1	(-1, 1)	q6
1	(-2, -2)	5	4	(-2, 1)	p2	(1, -1)	q7, q1
2	(-7/8, -2)	2.375	-6	(7/8, 1)	p3	(-1/8, -1)	q8, q9
3	(-1, -2)	-1	-7	(-1, 1)	p4	(0, -1)	None
4	(0, 2)	0	-17	(0, 1)	None	(-1, -1)	None

Figure 9. First Example of ETIS



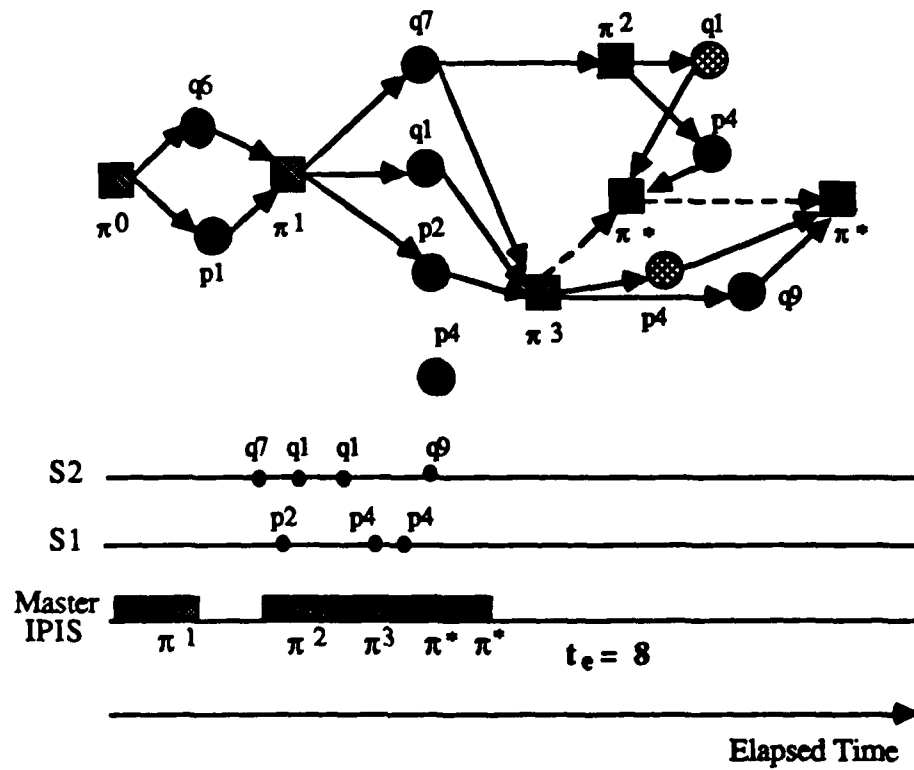
k	π	σ_1	σ_2	$(c_1 - \pi A_1)$	Q1	$(c_2 - \pi A_2)$	Q2
0	(0, 0)	0	0	(0, -1)	p_1	(-1, 1)	q_6
1	(-2, -2)	5	4	(-2, 1)	p_2, p_4	(1, -1)	q_7
2	(0, -2)	0	-13	(0, 1)	None	(-1, -1)	None

Figure 10. Second Example of ETIS

4.3 Examples of IPIS

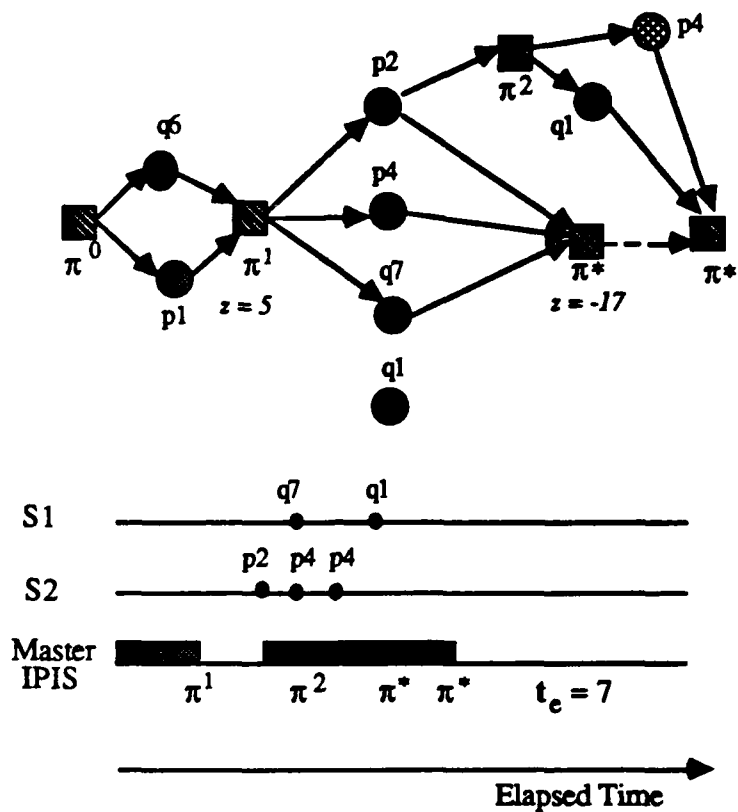
A proposals generation pattern and the resulting prices-proposals graph using IPIS (All-proposals) are shown in Figure 11. With this information scheme, the coordinator starts working as soon as q_7 is received. Two more proposals, generated with π^1 arrive while the coordinator is working. At the time π^2 is available to the subproblems, S1 is still busy while S2 is not. The arrival of prices π^2 starts S2, which generates q_1 at optimality. Note that q_1 has already been generated with π^1 . We call q_1 a repeated proposal. As soon as S1 receives π^2 it stops using π^1 which so far has generated p_2 . At optimality of S1 under π^2 , p_4 is obtained. Similarly, π^3 causes q_9 of S2 and p_4 of S1 to be generated. Note that p_4 is again a repeated proposal. The elapsed time for this example is approximately 8 units.

The behavior of IPIS assuming a different proposals generation pattern is shown in Figure 12. Note that repeated proposals are again involved. These examples demonstrate that the more frequent communication of prices may not necessarily speed up the distributed decision process. The intermediate prices definitely increase the volume of information transmitted among the agents. Portions of the additional information may be extraneous, e.g. in the form of repeated proposals. Whether the benefits of intermediate prices outweigh their cost again depends on the relative response times of the agents.



k	π	σ_1	σ_2	$(c_1 - \pi A_1)$	Q1	$(c_2 - \pi A_2)$	Q2
0	(0, 0)	0	0	(0, -1)	p_1	(-1, 1)	q_6
1	(-2, -2)	5	4	(-2, 1)	p_2	(1, -1)	q_7, q_1
2	(-2, -2)	5	3	(-2, 1)	p_4	(1, -1)	q_1
3	(-7/8, -2)	2.375	-6	(7/8, 1)	p_4	(-1/8, -1)	φ
4	(0, -2)	0	-17	(0, 1)	None	(-1, -1)	None

Figure 11. First Example of IPIS



k	π	$\sigma 1$	$\sigma 2$	$(c1-\pi A1)$	Q1	$(c2-\pi A2)$	Q2
0	(0, 0)	0	0	(0, -1)	p_1	(-1, 1)	q_6
1	(-2, -2)	5	4	(-2, 1)	p_2, p_4	(1, -1)	q_7
2	(-2, -2)	-1	4	(-2, 1)	p_4	(1, -1)	q_1
3	(0, -2)	0	-17	(0, 1)	None	(-1, -1)	None

Figure 12. Second Example of IPIS

5. Conclusions

For distributed decision processes that can be modeled as the decomposition of linear programs, we have shown that the dynamics of information among the agents play a crucial role in the effectiveness of particular strategies or policies. Four classes of decision policies are discussed formally as information schemes. In the basic scheme, the coordinator sets prices and then the planners submit proposals accordingly. When all the proposals are in, the coordinator gets to work again to determine the next round of prices. The relative timing of the proposals, which reflects perhaps individual work style or other behavioral aspects of the planners, does not matter in this case. The early-start scheme has the coordinator going back to work as soon as proposals become available. Meanwhile, the planners continue to submit proposals based on the last set of prices. The same sequence of prices as in the basic scheme is obtained. Potential advantage comes from the fact that the coordinator is getting a head start. Of course, it is also possible that the partial information used in such an early start may mislead the coordinator, in which case he might be better off waiting for the entire batch of proposals. As to the planners, uneven response times mean that some of them waste a lot of time waiting for others to finish their round of proposals. The early-termination scheme has all of them stopping at some threshold time, e.g. when the first planner finishes. They then wait for the coordinator to set new prices. How well this works compared to the basic scheme depends largely on the information omitted due to the early termination. Examples are shown for both cases. How about more information and sooner? This can be accomplished by extending the early-start scheme. Working with proposals as they arrive, the coordinator can revise the prices constantly. Should these prices replace the old ones at once? Intuitively, the idea of instant feedback of information and a decision process that approaches continuous adjustment has certain appeal. In reality, the burden of such excess of information and communication may be difficult to overcome. Again, examples are given to illustrate this point concerning such intermediate-prices schemes. Our study shows that it is not meaningful to look for a universally superior information scheme. However, using our approach of observing behavioral patterns of the coordinator and planners to the study of the dynamics of information among them, it may be possible to identify effective schemes for specific decision systems and conversely, to classify distributed decision systems by their most favorable schemes.

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communicated among the parties involved. We discuss four classes of information schemes and study the dynamics of information involved. It is shown that the overall effectiveness of any scheme depends on behavioral patterns of the interacting agents and that no single scheme can be universally superior.